

# SPECTRAL MEASUREMENTS OF TURBULENT HEAT AND MOMENTUM TRANSFER IN FULLY DEVELOPED PIPE FLOW

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**Abstract**—Cross-spectra of turbulent heat and momentum transfer in fully developed pipe flow of air are used to examine the similarity between the two processes. Results taken at a Reynolds number (based on pipe radius and centre line velocity) of 34 700 indicate that the low wavenumber components which in a previous study showed a high correlation between heat and momentum, are very ineffective in the transfer of these quantities from one fluid layer to the next in the wall region but become increasingly effective in the core flow. The mechanisms of the two transfer processes as measured by the spectral cross-correlation coefficients are found to be very similar in the energy containing range of wavenumbers. Local isotropy is approached by the velocity field at lower wavenumbers than by the temperature field.

## NOMENCLATURE

<p><math>a</math>, radius of tube = 2.67 in.:</p> <p><math>c_p</math>, specific heat at constant pressure [Btu/lb °F]:</p> <p><math>C_{uv}(k_1)</math>, fraction of energy of <math>\overline{uv}</math> associated with <math>k_1 = \frac{\overline{u(k_1) v(k_1)}}{\overline{uv}}</math>:</p> <p><math>C_{v\delta}(k_1)</math>, fraction of energy of <math>\overline{v\delta}</math> associated with <math>k_1 = \frac{\overline{v(k_1) \delta(k_1)}}{\overline{v\delta}}</math>:</p> <p><math>E_2(k_1)</math>, fraction of energy of <math>\overline{v^2}</math> associated with <math>k_1 = \frac{\overline{v^2(k_1)}}{\overline{v^2}}</math>:</p> <p><math>f</math>, friction factor:</p> <p><math>k_1</math>, one-dimensional wavenumber, [ft<sup>-1</sup>]:</p> <p><math>q''_w</math>, wall heat flux [Btu/h °F ft<sup>2</sup>]:</p> <p><math>Re</math>, Reynolds number based on bulk velocity and tube diameter:</p> <p><math>Re_{0a}</math>, Reynolds number based on velocity at centre line and radius of tube:</p> <p><math>R_{uv}</math>, total cross-correlation coefficient between <math>u</math> and <math>v</math>:</p> <p><math>R_{v\delta}</math>, total cross-correlation coefficient between <math>v</math> and <math>\delta</math>:</p> <p><math>R_{uv}(k_1)</math>, spectral cross-correlation coefficient between <math>u</math> and <math>v</math> at <math>k_1 = \frac{\overline{u(k_1) v(k_1)}}{\sqrt{[\overline{u^2(k_1)}][\overline{v^2(k_1)}]}}</math>:</p> <p><math>R_{v\delta}(k_1)</math>, spectral cross-correlation coefficient</p>	<p>between <math>v</math> and <math>\delta</math> at <math>k_1 = \frac{\overline{v(k_1) \delta(k_1)}}{\sqrt{[\overline{v^2(k_1)}][\overline{\delta^2(k_1)}]}}</math>:</p> <p><math>t</math>, static temperature [°F]:</p> <p><math>t_{\min}</math>, minimum static temperature at a given value of <math>x</math> [°F]:</p> <p><math>t_w</math>, wall temperature [°F]:</p> <p><math>t_{\tau}</math>, friction temperature = <math>q''_w / \rho c_p U_{\tau}</math> [°F]:</p> <p><math>u</math>, longitudinal velocity fluctuation (positive in stream direction) [ft/s]:</p> <p><math>U</math>, local mean velocity [ft/s]:</p> <p><math>U_{\tau}</math>, friction velocity = <math>\overline{U} \sqrt{f/2}</math> [ft/s]:</p> <p><math>\overline{U}</math>, bulk velocity [ft/s]:</p> <p><math>v</math>, normal velocity fluctuation (positive towards the wall) [ft/s]:</p> <p><math>x</math>, stream-wise distance [ft]:</p> <p><math>y</math>, radial distance from wall [in.]:</p> <p><math>y^+</math>, non-dimensional distance from wall = <math>yU_{\tau}/\nu</math>.</p> <p><b>Superscript</b>  <math>\bar{\phantom{x}}</math>, denotes time averaging.</p> <p><b>Subscripts</b>  <math>a</math>, pipe radius:  <math>u, v</math>, velocity fluctuation:  <math>\delta</math>, temperature fluctuation:  <math>0</math>, condition at centre line.</p>
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## Greek letters

$\Gamma(k_1)$ ,	fraction of energy of $\bar{\delta}^2$ associated with $k_1$ , $= \bar{\delta}^2(k_1)/\bar{\delta}^2$ ;
$\delta$ ,	stream temperature fluctuation [ $^{\circ}\text{F}$ ];
$\Delta T$ ,	$t_w - t$ [ $^{\circ}\text{F}$ ];
$\varepsilon$ ,	rate of viscous dissipation;
$\eta$ ,	Kolmogorov microscale;
$\eta_\theta$ ,	temperature field microscale;
$\nu$ ,	kinematic viscosity [ $\text{ft}^2/\text{s}$ ];
$\rho$ ,	mass density [ $\text{lb}/\text{ft}^3$ ].

## INTRODUCTION

THE CALCULATION of turbulent heat transfer has been a challenging problem for many years. Reynolds analogy which assumes similarity between heat and momentum transfer has been applied successfully in simple cases such as fully developed turbulent flow with Prandtl number near unity. Although this analogy yields reasonable engineering results it reveals no details concerning the physical processes and until a better understanding of these is available no great improvements in heat-transfer calculations can be expected.

Two distinct pieces of information are required. Firstly, an appreciation of the structure of the velocity field must be obtained and secondly, the inter-relation between heat and momentum transfer must be understood. In order to determine some aspects of the latter process, measurements of the relationship between the velocity and temperature fluctuations were made in fully developed, turbulent pipe flow. Those concerning the relationship between the longitudinal velocity and temperature fluctuations have already been reported [1] where it was shown that the low wavenumber components are highly correlated whereas at higher wavenumbers the correlation reduces rapidly with increase in wavenumber. Thus at the low wavenumbers the velocity and temperature fields are almost identical. The dominant process, therefore, is one wherein the low momentum fluid coming from the heated wall is also

hot fluid. This in turn displaces high momentum, cold fluid which, if it convects towards the wall, will result in the reverse process. If the two processes occur at almost all times then a near perfect correlation between heat and momentum is obtained as is the case at the low wavenumbers. At the higher wavenumbers the process is considerably less organized so that high momentum fluid is not always cold fluid and vice versa, thus resulting in a considerably lower correlation between longitudinal velocity and temperature fluctuations.

Although the flow processes are more random at the higher wavenumbers, mathematical predictions of the velocity and temperature spectra are more straightforward. A comprehensive survey and extensions of spectral forms in the universal range of such spectra are available [2, 3]. Except for the work of Tschen [4] for velocity fluctuations, no comparable predictions exist for the lower wavenumber ranges which actually contain the majority of the turbulent energy and hence can also be expected to make the major contribution to the turbulent diffusion processes. Predictions in the energy containing range of the spectrum are more difficult because of the larger number of variables which influence the processes. A useful step is, therefore, an experimental investigation of aspects of the relationship between the turbulent velocity and temperature fields, especially, in the energy containing range of wavenumbers. It is the aim of this paper to present such data, for fully developed pipe flow with constant heat flux, the emphasis being on the turbulent momentum and turbulent heat transfers directly. This involves the normal velocity fluctuation which should be of prime interest, since, in a parallel flow, it is the only flow component which can convect heat or momentum from one stratum of fluid to another.

Data examined include spectra of the longitudinal ( $u$ ) and normal ( $v$ ) velocity fluctuations, fluid temperature fluctuations ( $\delta$ ), cross-spectra of  $uv$  and  $v\delta$  and total and spectral cross-correlation coefficients  $R_{uv}$ ,  $R_{v\delta}$ ,  $R_{uu}(k_1)$ ,  $R_{v\delta}(k_1)$

where  $k_1$  is the conventionally defined one-dimensional wavenumber based on angular frequency and local mean velocity. The measurements were made with the lowest possible heat flux so that the commonly assumed flow condition of no change in the velocity field under heat addition would be approximated as closely as possible. Also, all fluctuating velocity and temperature data were taken simultaneously so that any changes in the velocity field would be detected and not lead to a false interpretation when comparing say  $uv$  and  $v\delta$  cross-spectra since the former could, of course, be obtained under conditions of no heat addition.

Detailed data are presented only for  $Re = 54500$  ( $Re_{oa} = 34700$ ) but similar results were obtained at  $Re = 148000$  ( $Re_{oa} = 91500$ ). The velocity and temperature signals and experimental conditions are the same as reported previously [1], but now include the normal velocity fluctuation which was omitted from the previous data.

#### EXPERIMENTAL APPARATUS

The working section of the wind tunnel consisted of a horizontally mounted 5.34 in. i.d., 0.375 in. wall thickness, 29 ft 8 in. long extruded aluminium tube through which air was forced by a centrifugal fan. A settling chamber designed to eliminate swirl and large scale turbulence generated by the fan and associated diffuser, separated the latter from the tube. In order to promote flow development a screen and boundary layer trip were placed at the entrance to the tube. The last 17 ft 2 in. of the tube were heated with 0.125 in. dia heating cable wound around the outside of the tube into grooves machined for the purpose.

Traverses were performed 1 ft 9 in. from the downstream end of the tube by introducing probes radially through the tube wall. The critical dimensions were, therefore, 28 tube diameters of unheated flow and 35 tube diameters of heated flow which, as expected from available literature, was shown to produce fully

developed flow at the traversing plane. The heating method used produced uniform heat flux which in turn gave a linear wall temperature gradient. A guard heater and appropriate thermal insulation were also provided to ensure no heat losses to the surroundings. For the low heat inputs of the present experiments a.c. heating was used the heat input being measured with a precision wattmeter.

#### INSTRUMENTATION

Hot wire anemometers were used as sensors in a three wire array which consisted of a conventional X-meter and a third wire placed at right angles to the plane of the X-meter. The X-wires were operated at the highest temperature which still gave rigid wires (thermal expansion at high wire temperatures can be quite serious). These wires, termed velocity wires, responded predominantly to velocity fluctuations. The third wire, the temperature wire, was operated at a very low wire current and, therefore, responded mainly to temperature fluctuations. This wire was placed 0.012 in. upstream of the geometric centre of the X-meter.

In order to minimize any interference from the temperature wire on the X-meter and also to get the best possible signal to noise ratio at the low wire temperature, a 0.0001 in. dia  $\times$  0.056 in. tungsten wire with copper plated ends was used. At the velocities reported, the wire Reynolds number for this wire was less than 1.3 and that of the plated ends less than 10. This was found to give negligible flow interference. Lateral separation between the inclined wires of the X-meter was 0.015 in. The velocity wires were of 0.0004 in. dia  $\times$  0.055 in. platinum-rhodium-ruthenium alloy (Sigmund-Cohn type 851) which is extremely stable both dimensionally and electrically.

The anemometer units consisted of Flow Corporation Model HWB constant current bridges each followed by a DC offset network and a Preston Model 8300 high gain, low noise, wide band, d.c. coupled, differential amplifier.

The normal open loop compensation for the time constant of each wire was achieved with a combination of EAI 231R analogue computer amplifiers and Tektronix type 3A8 operational amplifiers used for low noise, wide bandwidth differentiation.

For signal r.m.s. values or cross-correlation measurements, squaring or multiplication and integration were carried out on the analogue computer, which allowed any desired integration time to be used. The above signal amplification system gave exceptionally good long term stability with minimum signal distortion.

Spectral measurements were obtained by recording signals proportional to  $u$  and  $\delta$  on a Tolana FM tape recorder. Two Bruel and Kjaer type 2107 constant percentage bandwidth analysers were used for filtering the signals but signal squaring and averaging was again performed on the computer. In order to cover the desired frequency range it was necessary to record the signals at two different speeds—once at 15 i.p.s. and then at  $1\frac{7}{8}$  i.p.s. Replay was always at 15 i.p.s. For the long integration times and filter widths used, the normalized standard deviation of spectral results was 10 per cent at the lowest frequency and decreased to 1–2 per cent at the highest frequency. The frequency response of the combined hot wire amplification and signal processing equipment was flat to within 2 per cent up to 2 kHz which was quite adequate. A simple R–C filter with a 2 s time constant was used to remove any d.c. levels prior to signal processing. No error was produced by phase effects as care was taken to use phase matched signal channels. Analyser noise bandwidths were 10.5 per cent of the centre frequency.

The correction procedure applied to the hot wire anemometer signals in order to obtain true velocity and temperature signals is outlined in [1].

#### EXPERIMENTAL RESULTS

Radial temperature profiles agreed well with

available published ones, but as seen from Fig. 1(a) a small amount of distortion of the profile due to buoyancy was encountered. Friction factors as obtained from axial wall pressure drop measurements in isothermal flow agreed with the Blasius relationship which was used for all subsequent calculations of  $U_\tau$ . Measured Nusselt numbers agreed with the Dittus and Boelter correlation.

Longitudinal and normal velocity and stream temperature fluctuation intensities are shown in Fig. 1(b). It is seen that velocity fluctuation intensities for the unheated and heated flows agree well indicating that the small amount of buoyancy did not seriously alter the flow pattern.

Turbulent momentum transfer ( $uv$ ) and  $R_{uv}$  results are shown in Figs. 2 and 3 for both unheated and heated flows. The calculated curve is based on the velocity distribution in the unheated flow. It is seen that, for the unheated flow, measured and calculated momentum transfers agree except near the wall where spatial resolution due to wire separation and wire length has reduced the measured values. Effects due to the latter can be readily assessed [5] and results indicate that, although  $\overline{uv}$  is reduced considerably, the correlation coefficient,  $R_{uv}$ , is not affected to the same extent. The data for the heated flow show some deviation from the expected profiles but this is due to flow distortion by the added heat. This is readily seen from the measurement at the centre line where zero correlations were obtained in the unheated flow but finite values in the heated flow. Comparison with mean velocity profile data has shown that the zero value of  $\overline{uv}$  and hence  $R_{uv}$  does, however, still occur at the point of zero slope of the mean profile. The fact that a zero correlation was obtained at the pipe centre line for the unheated flow also means that accurate separation of the  $u$  and  $v$  velocity components from the X-wire signals was obtained.

Measured values of the correlation coefficient between the normal velocity and temperature

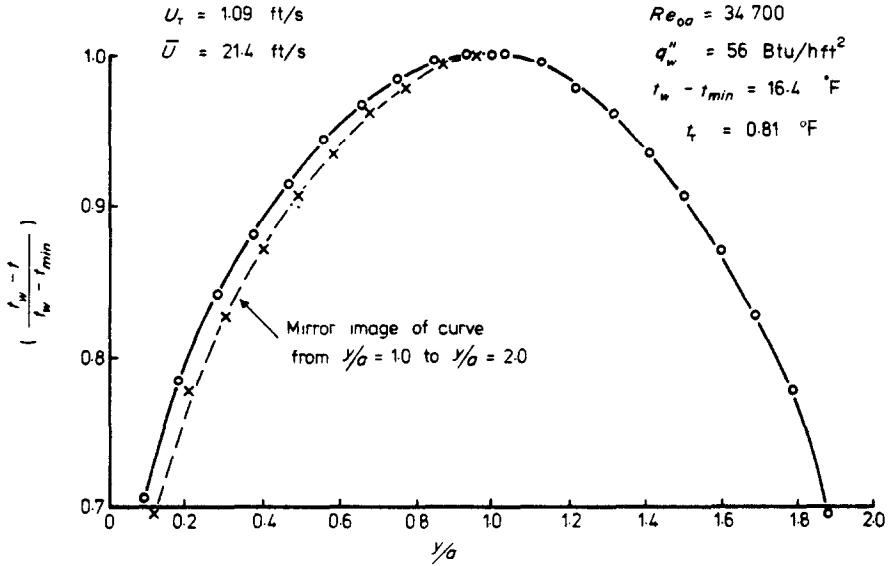


FIG. 1(a). Typical radial temperature profile.

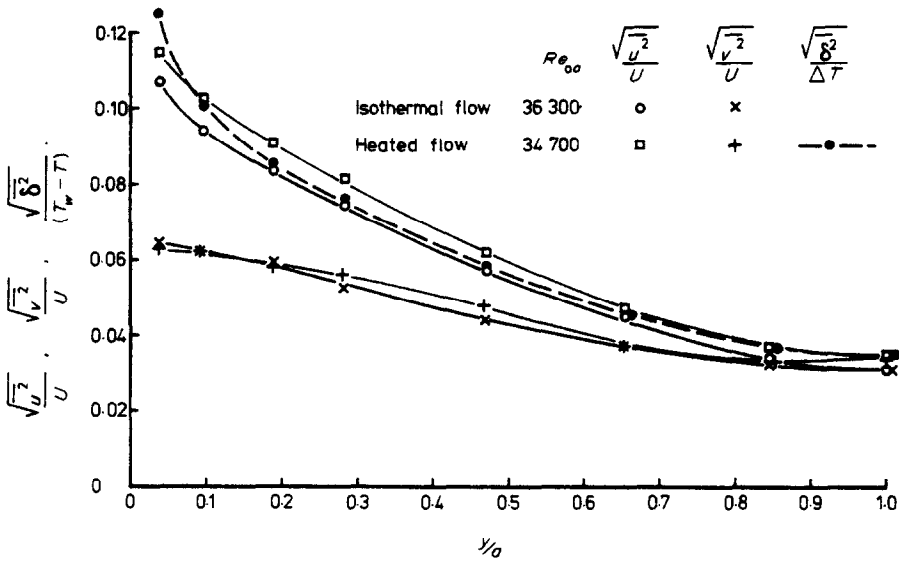


FIG. 1(b). Intensity of temperature and velocity fluctuations.

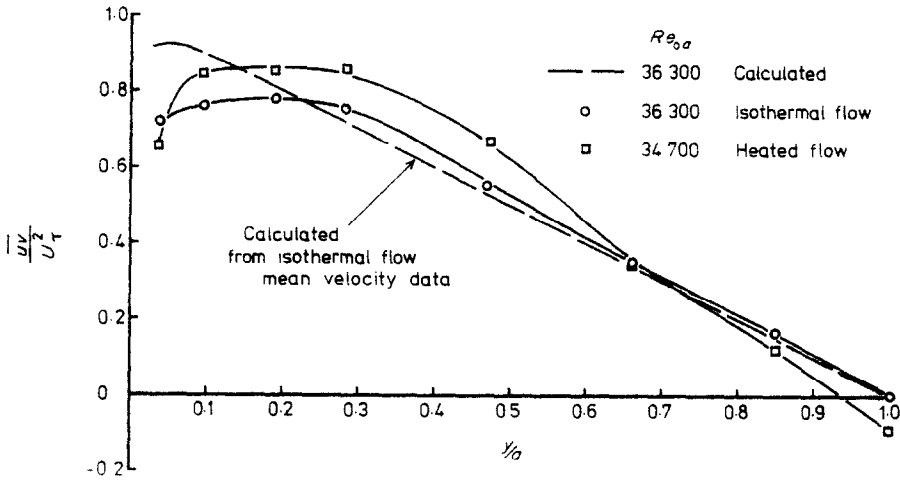


FIG. 2. Comparison of measured and calculated radial momentum transfer.

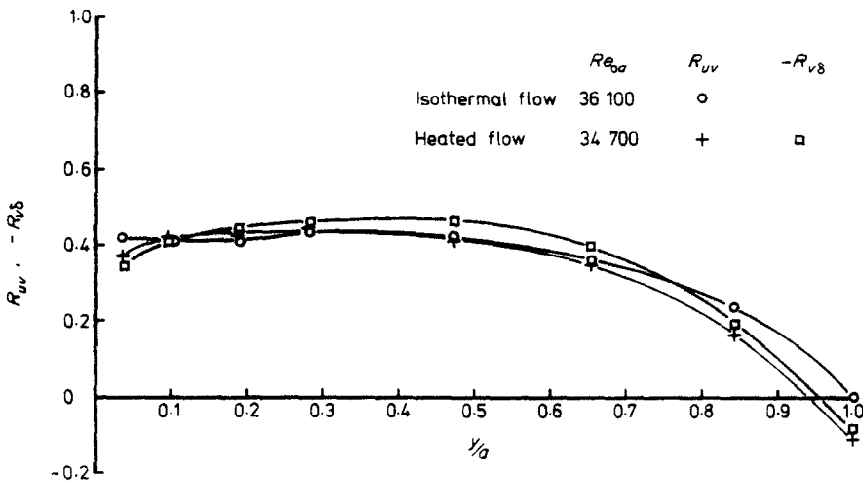


FIG. 3. Turbulent momentum and heat transfer correlation coefficients.

fluctuations are also shown in Fig. 3. These are remarkably similar to  $R_{uv}$  which indicates a high degree of similarity between heat and momentum transfer and agree with the data of Bourke and Pulling [6].

**SPECTRA OF NORMAL VELOCITY AND TEMPERATURE FLUCTUATIONS**

Presentation of spectra follows the method of [1], viz., the use of linear-log plots rather than the log-log plots which are frequently used

in turbulence work. It should also be noted that this method of plotting spectra is extremely sensitive to small changes compared with the generally used log-log plot.

Spectra of the normal velocity fluctuations for the unheated and heated flows are shown in Fig. 4 for various radial positions in the pipe. It is seen that the spectra for the two flow situations are very similar except for a consistent shift of energy to the lower wavenumber range. This is attributed to the action of buoyancy in the flow which would be expected to introduce low wavenumber components resulting in a slight distortion of the spectra in the manner shown. Identical trends were found with the spectra of longitudinal velocity fluctuations [1]. The spectra are normalized so that the small changes in total fluctuation energy (refer Fig. 1(b)) do not confuse the spectral shifts. Corresponding spectra of temperature fluctuations are also shown in Fig. 4. The latter are quite different from those of the normal velocity fluctuations but as found in [1] are very similar to the spectra of the longitudinal velocity fluctuations as would be expected from any analogy between heat and momentum transfer. This will be further emphasized by the spectral correlation coefficients.

The significant results are that the bulk of the normal velocity fluctuation energy is concentrated at higher wavenumbers than that of the temperature fluctuations and that the normal velocity fluctuation spectra are almost identical and single peaked at all radial positions, whereas the temperature spectra show a distinctive change from the core region through the log-layer and towards the edge of the buffer layer.

#### CROSS-SPECTRAL RESULTS

Two types of cross-spectral data are considered—the cross-spectra  $C_{uv}(k_1)$ ,  $C_{v\delta}(k_1)$  and the cross-spectral correlation coefficients  $R_{uv}(k_1)$  and  $R_{v\delta}(k_1)$ . From the cross-spectra it is possible to assess in which wavenumber range the majority of the turbulent transport takes place.

It is this region which is of interest from an engineering viewpoint. Spectral correlation coefficients, on the other hand, give information regarding the transfer mechanism. The two quantities are related but depend on the velocity and/or temperature fluctuation spectra so that a high correlation of two spectral components does not necessarily mean that a large amount of momentum or heat transfer takes place at this wavenumber. The cross-correlation coefficient can, in fact, be thought of as an efficiency of mixing or degree of interaction between layers of fluid of different velocity or temperature.

Cross-spectra of turbulent momentum and heat transfer at various radial positions are shown in Fig. 5. Included are values of  $C_{uv}(k_1)$  measured in the unheated flow to show that heat addition has altered the flow to a small extent. The results show that the cross-spectra of momentum and heat are quite similar at most points in the flow. The major contributions to both processes are in the same wavenumber regime and most important perhaps is the fact that the processes occur over a considerable range of wavenumbers rather than in a narrow band. From these results it could be hypothesized that heat and momentum are transferred in an identical manner. However, this is incorrect since the only aspect which has been shown to be similar is the contribution each wavenumber band makes to the total turbulent transfer.

Spectral correlation coefficients are shown in Fig. 6(a). Several significant trends are observed. In the bulk of the flow a steady decrease in cross-correlation for both  $u-v$  and  $v-\delta$  occurs as wavenumber increases. This is consistent with the generally accepted notions that feeding from the mean flow involves only the larger eddies which in turn feed energy to the smaller ones so that anisotropy occurs at low wavenumbers and isotropy is approached at high wavenumbers with a consequent vanishing of spectral correlation coefficients. Near the wall the opposite trend is found. Although measurements were taken only to  $y^+ = 52$ , the trends

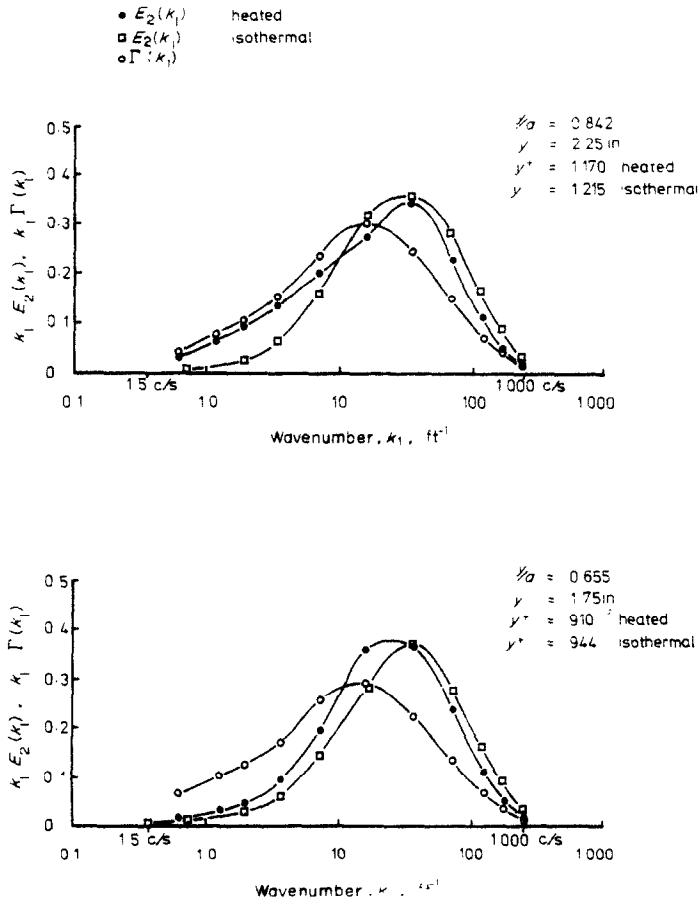


FIG. 4(a). Spectra of normal velocity and temperature fluctuations.

clearly show that if measurements were taken even closer to the wall the correlation coefficient of the low wavenumber components would decrease even further whereas correlation coefficients of the higher wavenumber components would remain at significant values probably right to the sublayer. Thus the large flow components (small wavenumbers) have a small degree of interaction between fluid layers of different velocities or temperatures whereas the smaller scale components show the opposite trend. Extrapolation of these results and the

spectral data of Figs. 4 and 5 indicate that as the wall is approached, the contribution of the low wavenumber components, as measured by the cross-spectra  $C_{u\delta}(k_1)$  and  $C_{v\delta}(k_1)$ , decreases markedly towards the wall with the result that the transport processes take place over a much smaller wavenumber range than may be assumed from  $u, v$  and  $\delta$  spectra alone. This extrapolation has, in fact, already been verified [7] for turbulent momentum transfer in fully developed, isothermal flow and since the correlation between  $u$  and  $\delta$  increases towards the wall [1] it



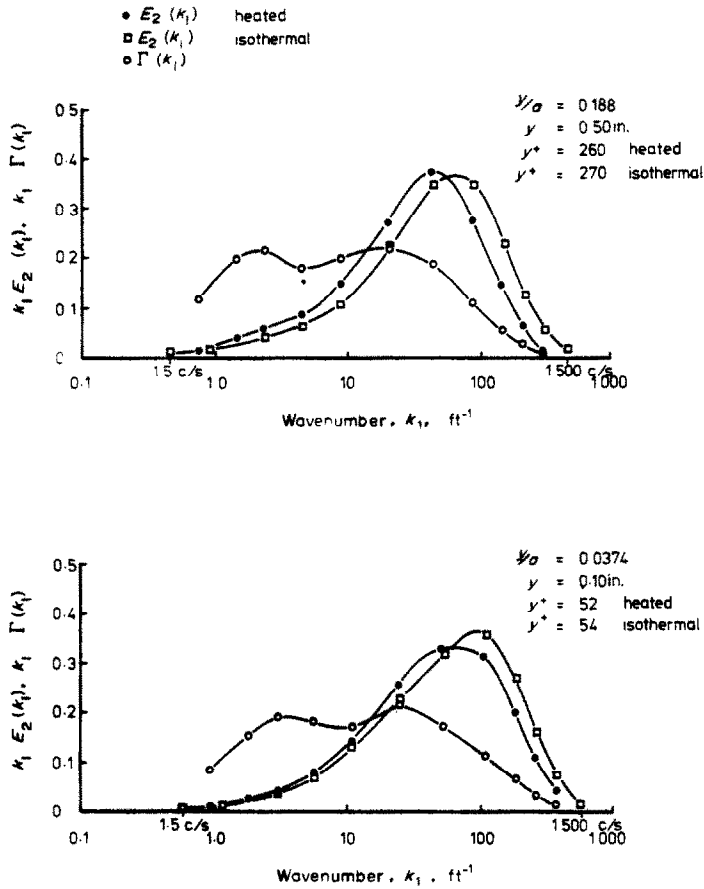


FIG. 4(b). Spectra of normal velocity and temperature fluctuations.

follows that the extrapolation also holds for non-isothermal flow as suggested.

Figure 6(b) shows a comparison of spectral momentum cross-correlation coefficients as measured in the unheated and heated flows. The distortions at the centre of the pipe and at the pipe wall at very low wavenumbers are again noted. In the essentially undisturbed region,  $0.2 < y/a < 0.7$ , the two sets of data agree well.

The other important aspect of the results is that at the higher wavenumbers,  $R_w(k_1)$  and

$R_{\theta\theta}(k_1)$  are still quite high even though the turbulent energy content as seen from the spectra of the individual turbulence components is small. Isotropy is, therefore, approached only very slowly and certainly does not exist in the wavenumber range examined. Although microscales were not measured, the present spectral data can be placed in perspective with the aid of established empirical flow parameters [8] and [9] which are shown in Table 1 for the present case.

Table 1. Viscous and conductive cut-offs,  $Re_{\sigma_s} = 34\,700$

$y/a$	$y^+$	$\epsilon a/U_\tau^3$	$k_1 = \frac{1}{\eta}$ (ft <sup>-1</sup> )	$k_{1\delta} = \frac{1}{\eta_\delta}$ (ft <sup>-1</sup> )
0.843	1170	1.08	1000	830
0.188	260	5.36	1600	1230

Since viscous and conductive effects become significant at  $0.1/\eta$  and  $0.1/\eta_\delta$ , respectively [8],

it is seen that for the flow investigated a very small to negligible universal spectral range can be expected as the energy containing range extends to at least  $k_1 = 50 \text{ ft}^{-1}$ . This is consistent with the existence of significant values of  $R_{uv}(k_1)$  and  $R_{v\delta}(k_1)$  even at  $k_1 = 100 \text{ ft}^{-1}$  (Fig. 6(a)). The values in Table 1 are therefore only approximate.

Although the conduction cut-off occurs at lower wavenumbers than the viscous cut-off, replotting the results of Fig. 6(a) as in Fig. 7 shows that the velocity field approaches local

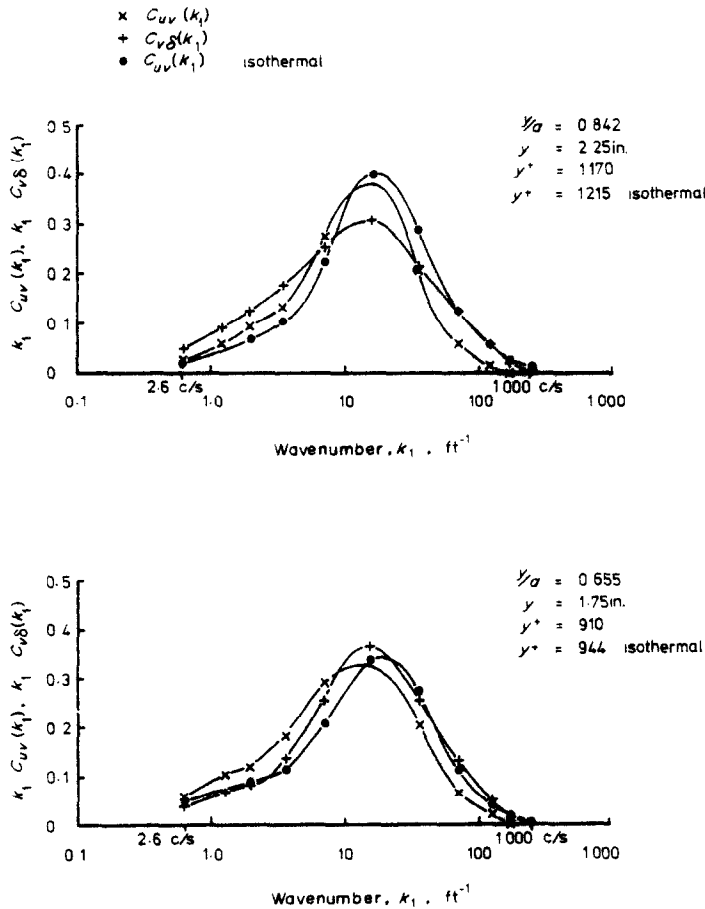


FIG. 5(a). Comparison of turbulent heat and momentum transfer cross-spectra.

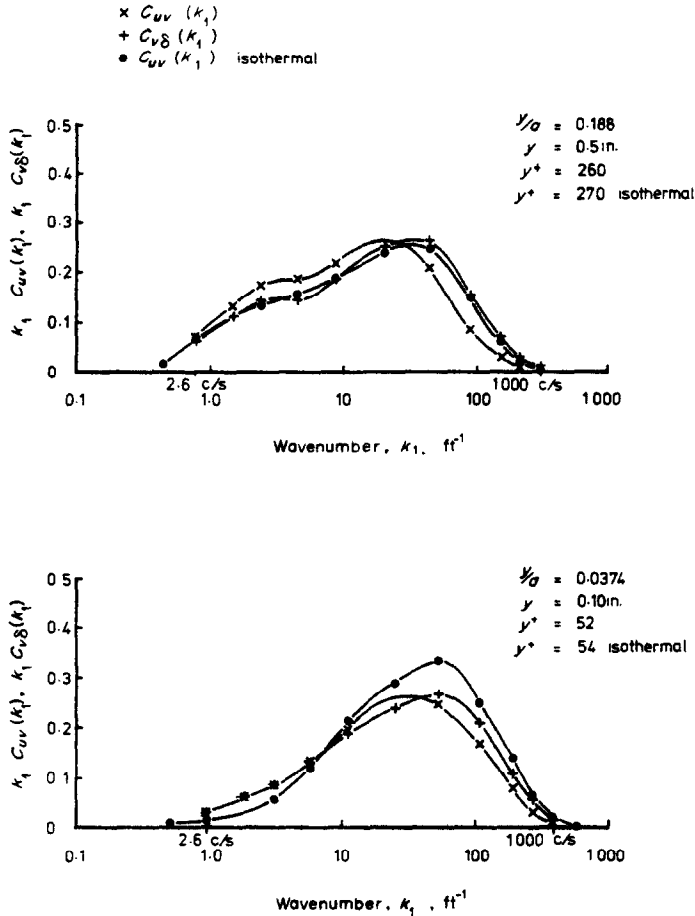


FIG. 5(b). Comparison of turbulent heat and momentum transfer cross-spectra.

isotropy at smaller wavenumbers than the temperature field. This is consistent with the  $u$  and  $\delta$  spectra [1] from which it is seen that the energy containing wavenumber range or range where interaction with the mean field takes place occurs at higher wavenumbers for the temperature field than for the velocity field. Figure 7 also shows that if the spectral correlation coefficients are accepted as a measure of the transfer mechanism or effectiveness, then heat and momentum are transferred almost identically in the wall region, particularly in the energy containing range of wavenumbers. How-

ever, heat is transferred more effectively than momentum in the core of the flow.

**OTHER REYNOLDS NUMBERS**

Similar measurements were made at  $Re = 148000$  ( $Re_{0a} = 91500$ ). The results followed those at the lower Reynolds number but slight flow interference from the temperature wire or its supports resulted in some errors in the velocity spectra. No measurements were made at lower Reynolds numbers than the one described but it is known from velocity spectra

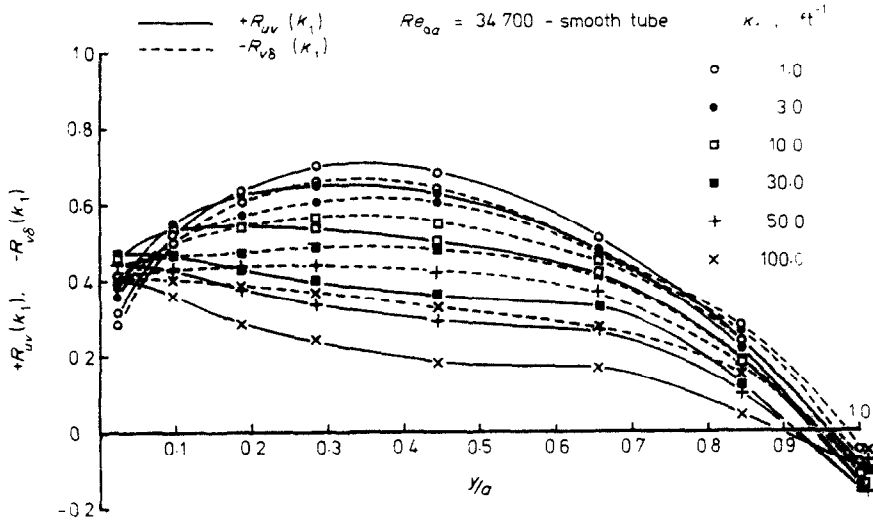


FIG. 6(a). Turbulent heat and momentum transfer correlation coefficients.

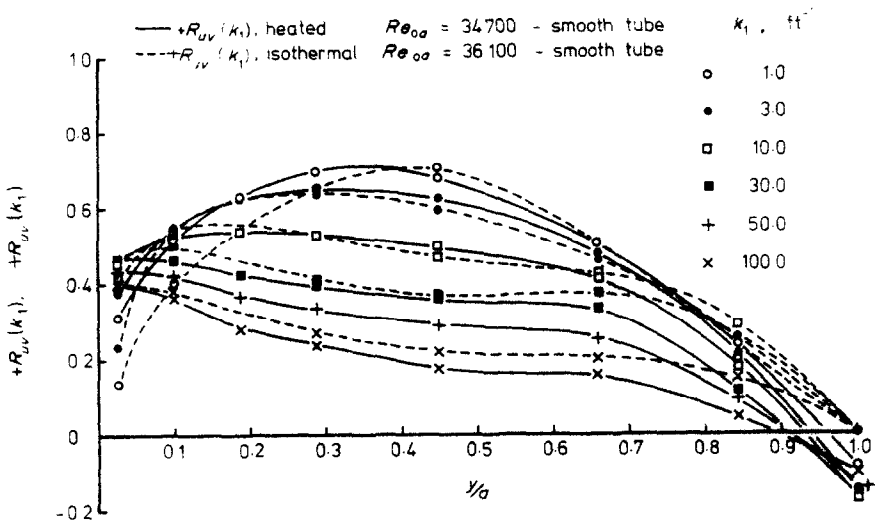


FIG. 6(b). Comparison of isothermal and non-isothermal momentum transfer correlation coefficients.

[10] that below  $Re = 22000$  ( $Re_{0a} = 17000$ ), spectra of the longitudinal velocity fluctuations do not exhibit the characteristic double peaks found at higher Reynolds numbers in the log-layer regions. Assuming that heat and momen-

tum transfer processes are still similar, it follows that the temperature fluctuation spectra will follow the same trend. Direct comparison of such lower Reynolds number flows with the present results is, therefore, not possible as

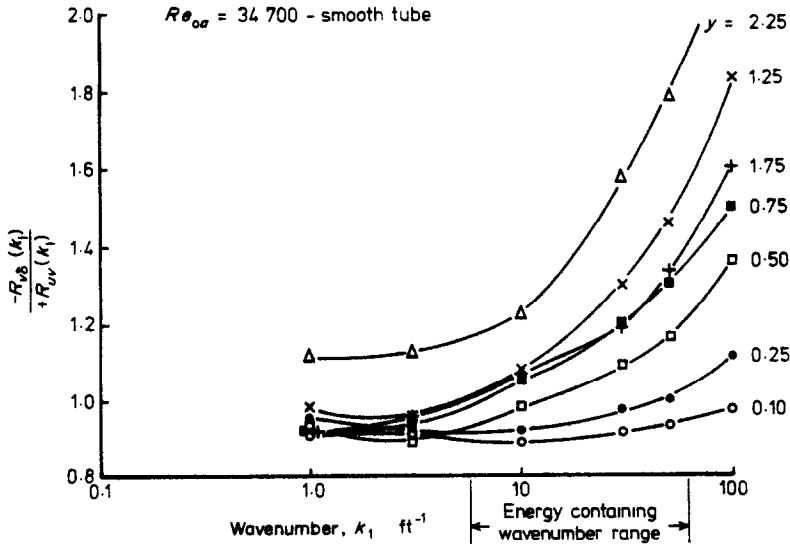


FIG. 7. Ratio of turbulent heat and momentum transfer spectral correlation coefficients.

cross-spectra are, undoubtedly, also affected by the change in flow behaviour. Cross-spectral correlation coefficients should, however, be unaffected but this is subject to experimental verification.

### CONCLUSIONS

Interpreting the spectral correlation coefficients,  $R_{vv}(k_1)$  and  $R_{v\theta}(k_1)$  as a measure of the flow mechanism, the results show that for fully developed pipe flow, heat and momentum are transferred identically in the energy containing range of wavenumbers of the wall regions but that in the core regions the heat transfer process is more effective.

The measurements illustrate that the dominant part of the turbulent transfer process occurs over a large range of wavenumbers. Spectral cross-correlation coefficient measurements also indicate that in the core of the flow a steady trend from a high degree of anisotropy to local isotropy takes place with increasing wavenumber. At the wall a significantly different situation exists. In this region the low wave-

numbers show very little correlation and the higher wavenumbers in the energy containing range tend towards higher correlations before reducing again at still higher wavenumbers. The relative roles or mechanisms of heat and momentum transfer are, however, unaffected.

From published empirical data it is shown that a universal wavenumber range could not be expected and this was confirmed by the spectral cross-correlation measurements. The latter also showed that local isotropy is approached at lower wavenumbers for the velocity field than for the temperature field even though the viscous cut-off occurs at higher wavenumbers than the conductive cut-off.

### ACKNOWLEDGEMENT

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### REFERENCES

1. K. BREMHORST and K. J. BULLOCK, Spectral measurements of temperature and longitudinal velocity fluctuations in fully developed pipe flow, *Int. J. Heat Mass Transfer* 13, 1313-1329 (1970).

2. G. K. BATCHELOR, Small-scale variation of convected quantities like temperature in turbulent fluid—Part I, *J. Fluid Mech.* **5**(1), 113–133 (1959).
3. G. K. BATCHELOR, I. D. HOWELLS and A. A. TOWNSEND, Small-scale variation of convected quantities like temperature in turbulent fluid—Part II, *J. Fluid Mech.* **5**(1), 134–139 (1959).
4. TSCHEN—refer J. O. HINZE, *Turbulence*, pp. 263–268. McGraw-Hill, New York (1959).
5. K. BREMHORST, The effect of wire length and separation on X-array hot wire anemometer measurements, *IEEE. Instr. Meas.* **IM-21** (3 Aug. 1972).
6. P. J. BOURKE and D. J. PULLING, A turbulent heat flux meter and some measurements of turbulence in air flow through a heated pipe, *Int. J. Heat Mass Transfer* **13**, 1331 (1970).
7. K. BREMHORST and T. B. WALKER, Spectral measurements of turbulent momentum transfer in fully developed pipe flow, Res. Report 5/72, Dept. of Mech. Eng., University of Queensland, Australia (1972).
8. P. BRADSHAW, *An Introduction to Turbulence and its Measurement*, pp. 38–39. Pergamon Press, Oxford (1971).
9. C. J. LAWN, The determination of the rate of dissipation in turbulent pipe flow, *J. Fluid Mech.* **48**(3), 477–505 (1971).
10. W. R. B. MORRISON, K. J. BULLOCK and R. E. KRONAUER, Experimental evidence of waves in the sublayer, *J. Fluid Mech.* **47**(4), 639–656 (1971).

### MESURES SPECTRALES DU TRANSFERT TURBULENT DE CHALEUR ET DE QUANTITÉ DE MOUVEMENT POUR UN ÉCOULEMENT ÉTABLI DANS UN TUBE

**Résumé**—On a procédé à des mesures spectrales de transfert turbulent de chaleur et de quantité de mouvement pour un écoulement d'air établi dans un tube afin d'étudier la similitude entre les deux mécanismes. Des résultats, obtenus à un nombre de Reynolds (basé sur le rayon du tube et la vitesse au centre) de 34 700, montrent que les composantes à faible nombre d'onde qui montraient dans une étude précédente une haute corrélation entre chaleur et quantité de mouvement, sont sans efficacité dans le transfert de ces quantités d'une couche fluide à l'autre près de la paroi, mais deviennent progressivement efficaces au coeur de l'écoulement. Les mécanismes des deux processus de transfert, mesurés par les coefficients de corrélation spectrale, sont très semblables dans la gamme des nombres d'onde contenant l'énergie. L'isotropie locale est mieux approchée par le champ des vitesses aux plus faibles longueurs d'onde que par le champ des températures.

### SPEKTRALMESSUNGEN DER TURBULENTEN WÄRME- UND IMPULSÜBERTRAGUNG IN VOLLAUSGEBILDETER ROHRSTRÖMUNG

**Zusammenfassung**—Kreuzspektre der turbulenten Wärme- und Impulsübertragung bei vollausgebildeter Rohrströmung für Luft werden herangezogen, um die Ähnlichkeit zweier Prozesse zu prüfen. Die bei einer Reynolds-Zahl (gebildet aus Rohrradius und Geschwindigkeit längs der Rohrmitte) von 34 700 erhaltenen Ergebnisse weisen darauf hin, dass die niedrigen Wellenzahl-Komponenten, welche in einer früheren Untersuchung eine grosse Korrelation zwischen Wärme und Impuls erkennen liessen, für die Übertragung dieser Grössen von einer Flüssigkeitsschicht zur nächsten in Wandnähe unwirksam sind, aber in der Kernströmung zunehmend wirksam werden.

Die Mechanismen der zwei Übertragungsprozesse, gemessen mittels der spektralen Kreuzkorrelationskoeffizienten, sind in den Energie enthaltenden Wellenzahl-Bereichen als sehr ähnlich gefunden worden. Das Geschwindigkeitsfeld erreicht lokale Isotropie bei kleineren Wellenzahlen als das Temperaturfeld.

### СПЕКТРАЛЬНЫЕ ИЗМЕРЕНИЯ ТУРБУЛЕНТНОГО ПЕРЕНОСА ТЕПЛА И ИМПУЛЬСА В ПОЛНОСТЬЮ РАЗВИТОМ ТЕЧЕНИИ В ТРУБЕ

**Аннотация**—Поперечные спектры турбулентного переноса тепла и импульса в полностью развитом потоке воздуха в трубе используются для исследования подобия этих двух процессов. Результаты для числа Рейнольдса, равного 34 700 (характерными размерами являются радиус трубы и скорость на оси), показывают, что компоненты этого спектра для низких волновых чисел, которые в предыдущей работе указывали на тесную связь тепла и импульса, не играют существенной роли при переносе этих субстанций от одного слоя жидкости к другому в пристенной области; однако их влияние значительно возрастает по направлению к ядру потока. При измерении спектральных коэффициентов взаимной корреляции установлено, что механизмы этих двух процессов весьма подобны в энергосодержащей области волновых чисел. Локальная изотропия поля скорости достигается при более низких волновых числах по сравнению с полем температур.